Dynamic Modeling and Natural Frequency of a Multi-Crack Extensible Beam under Vertical Load

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수직 하중을 받는 다중 균열 신장 보의 동적 모델링과 고유 진동수

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요 약

The Euler-Bernoulli beam theory is widely utilized for the analysis of one-dimensional structures, particularly beams subject to bending due to gravitational forces. It provides a framework for describing beam deflections, stability, and vibration characteristics. This theory has been extensively applied to investigate the natural frequencies of beams with multiple cracks, where cracks are often modeled as virtual springs. However, the classical Euler-Bernoulli beam theory neglects the influence of axial deformation. In this study, we develop a dynamic model for cracked extensible beams, incorporating the effects of axial deformation, and examine the resulting changes in natural frequencies. The axial force induced by deflection significantly alters the beam's natural frequency range, and this relationship is further explored through numerical simulations.

1. The Extensible Beam with Multiple Cracks

The motion equation governing vibration of the multi-cracked beam under large deflection is as follow:

$$v_{,tt} + cv_{,t} + v_{,xxxx} - N\,v_{,xx} = p \tag{1}$$

$$J[v](x_0) = J[v_{,xx}](x_0) = J[v_{,xxx}](x_0) = 0 \tag{2}$$

$$J[v_{,x}](x_0) = \theta\, v_{,xx}(x_0^-) \tag{3}$$

$$NJ[v_{,x}](x_0) = 0 \tag{4}$$

$$v(0,t) = v(1,t) = v_{,xx}(0,t) = v_{,xx}(1,t) = 0 \tag{5}$$

where, $N = \frac{1}{2}\int_0^1 v_{,x}^2\,dx$, $\theta = \frac{h}{L}C(\beta)$

2. Eigenfunction Expansion of Governing Equation

Assume that the solution y and the load p can be expressed

$$v(x,t) = \sum_{j=1}^{\infty} Y_j(t)\,\phi_j(x) \tag{6}$$

$$p(x,t) = \sum_{j=1}^{\infty} q_j(t)\,\phi_j(x) \tag{7}$$

Here, the external force p(x,t), represented as a series function, consists of the eigenfunction and the coefficient. Following patching condition, we have

$$w_i^{\lambda_j}(x) = \overrightarrow{A}_i^j \cdot \vec{b}_i^j \tag{8}$$

Substituting the series for y and p into the governing equation, and taking the inner product, we obtain the governing equations using these inner product terms.

$$\ddot{Y} + c\dot{Y} + \lambda^4 Y - \frac{N}{\Phi}(A + A^\theta) = p \tag{9}$$

where,

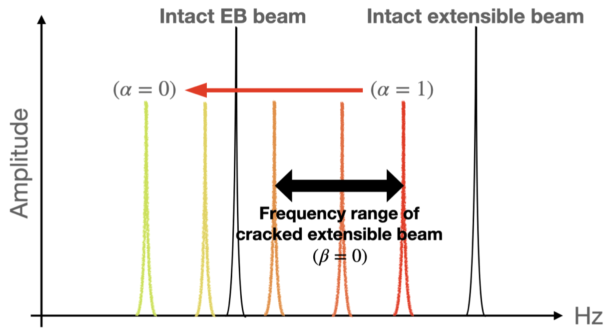
$$A = Y\lambda^2 \sum_{i=1}^{k+1} \int_{x_{i-1}}^{x_i} \left(\overrightarrow{A}_i \cdot \vec{b}_{i2}\right)\left(\overrightarrow{A}_i \cdot \vec{b}_i\right) dx$$

$$A^\theta = Y\lambda^2 \sum_{i=1}^{k} \theta_i \left(\overrightarrow{A}_i \cdot \vec{b}_{i2}(x_i)\right)\left(\overrightarrow{A}_i \cdot \vec{b}_i(x_i)\right)$$

In this governing equation, ignoring N makes the model linear, while $N \neq 0$ leads to a nonlinear system, indicating large deflections in the multi-crack extensible beam.

3. Natural Frequency of Crack Extensible Beam

The multi-cracked extensible beam model shows that increased deflection raises both axial force and frequency. Cracks reduce the frequency, similar to the effect in an EB beam, but the extensible beam has a higher frequency range due to tension from deflection. However, with deep cracks, the frequency can approach that of an intact EB beam.



[Fig. 1] Comparison of frequencies between intact beam and cracked beam, accounting for the axial force

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