A Study on the Static Analysis of Truss Using Force Method-Based Multi-Layer Neural Networks

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내력법 기반의 다중 신경망을 이용한 트러스 구조물의 정적 해석에 관한 연구

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요 약

This study proposes a neural network-based surrogate model for the static analysis of truss structures. The model utilizes the mapped index domain or member indices of the spatial coordinate domain as input, and predicts displacements or member forces as output for planar and spatial trusses with pin joints subjected to arbitrary loads. The training process of the neural network incorporates residuals along with equilibrium and compatibility conditions into the loss function. To validate the proposed DNN structure based on equilibrium and compatibility conditions, numerical analyses were conducted on a 10-bar planar truss and a 25-bar spatial truss. The results confirm the effectiveness of the proposed neural network model, with the accuracy of the analysis found to vary based on the choice of residual parameters.

1. Analysis Model of Truss Structures

The force method offers a simpler relationship between the displacement vector and internal force vector, enabling a more intuitive understanding of structures.

With the external load vector p and internal force vector t, the equilibrium equation is expressed as At=b. The compatibility between the displacement vector d and the elongation vector e is given by Bd=e.

The relationship between internal force and elongation for structures satisfying equilibrium and compatibility is represented by the flexibility matrix F as Ft=e.

The general solution t to the equilibrium equations includes a particular solution t_A, and the general solution is given by

$$t = t_A + S\alpha \tag{1}$$

Self-equilibrium and solving for α gives as

$$\alpha = -(S^T F S)^{-1}(S^T e_0 + S^T F t_A) \tag{2}$$

This result allows calculating the internal force, where e_0 accounts for initial strain and t_A for external load effects.

2. Equilibrium force informed Neural Network

In the model for the boundary value problem concerning the static analysis of truss structures, the data-driven loss function is generally as follows:

$$L_r = \frac{1}{N}\|\overline{u_S} - \hat{u}(x,\theta)_S\|_2^2 + \frac{1}{N}\|\overline{u_B} - \hat{u}(x,\theta)_B\|_2^2 + \lambda_{L_2}\|\theta\|_2^2 \tag{3}$$

Here, the subscripts S and B represent the spatial domain and boundary, respectively, $\bar{u}$ and $\hat{u}$ are the target data and predicted output, respectively, and λ_{L_2} is the regularization coefficient. While the nodal displacements can be divided into node and boundary, the output related to internal forces can be defined as $L_r = \frac{1}{N}\|\bar{t} - \hat{t}(x,\theta)\|_2^2 + \lambda_{L_2}\|\theta\|_2^2$

When this problem is restructured into a model that considers the physical constraints, including equilibrium (or compatibility) conditions of the structure, it is as:

$$\begin{aligned} &\min_{\theta} \quad L_r \\ &s.t.\ \ At = p \end{aligned} \tag{4}$$

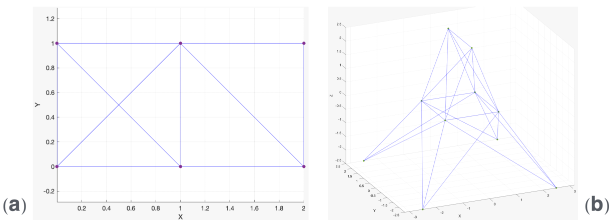
By using the information on the considered constraint functions and converting the problem into a Lagrangian duality framework with an added loss L_q of empirical risk function, it can be reformulated into the following max-min problem as follows:

$$\max_{\lambda \geq 0} \min_{\theta} \ L(x,\lambda,\beta,\theta) = L_r + \lambda L_L + \beta L_q \tag{5}$$

Here, λ and β represent the residual parameters, i.e., the Lagrange multipliers and the coefficients of the empirical risk function, respectively, and L_L is can be defined equilibrium matrix or compatibility matrix. Using the proposed framework, the residual parameters are updated sequentially to train the neural network model and obtain the solution.

3. Numerical Examples

Numerical analyses were conducted on a 10-bar planar truss and a 25-bar spatial truss.



[Fig. 1] Numerical Examples; (a) 10-bar planar truss, (b) 25-bar space truss

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (RS-2023-00248809, RS-2024-00413824).

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